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MONEY AND THE TERMS OF TRADE

Julio J. Rotemberg*

W.P. #1344-82

September 1982

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*Sloan School of Management
Massachusetts Institute of Technology

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ABSTRACT

This paper examines the connection between money and the terms of trade in the context of a simple monetary equilibrium model with flexible prices. Money is held for transactions purposes. Because carrying out financial transactions is costly households visit their financial intermediaries only occasionally. An important feature of the model is that different households visit the financial intermediaries at different times. This is sufficient to ensure that even though the model features perfect foresight and competitive markets, monetary policies affect output under both fixed and flexible exchange rates. Moreover, in the latter regime expansionary monetary policies tend to worsen the terms of trade while this is not the case under fixed exchange rates.

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I Introduction

This paper studies the connection between money and the terms of trade in the context of a simple monetary general equilibrium model with flexible prices. In this model money is held for transactions purposes. As in the models of Baumol (1952) and Tobin (1956) but in contrast to the open economy model of Stockman (1980) households are allowed to hold interest bearing capital in addition to barren money. In fact their portfolio includes claims on both foreign and domestic capital. Households pick consumption paths which maximize their utility. Because it is costly to carry out financial transactions they engage in these transactions only sporadically. However households are not allowed to time these transactions optimally. Instead, for tractability, I assume the length of the period during which no financial transactions are carried out is constant.

However, as in the models of Grossman and Weiss (1982) and Rotemberg (1982) as well as in all actual free market economies, different people visit their financial intermediaries at different times. This assumption is crucial to make monetary policy affect the terms of trade. This occurs even though the model features competitive markets and perfect foresight. In this model, under flexible exchange rates, as long as the technologies in the two countries are similar expansionary open market operations tend to worsen the terms of trade. This, as can be seen in Branson (1979), is also true in the simple textbook models with capital mobility. However, it stands in sharp contrast to the neutralities

reported by Stockman (1980) and, for the money in the utility function model, by Obstfeld (1981). This non-neutrality can be explained as follows. When money increases in the home country, it tends to raise prices at home. This reduces the purchasing power of the money held by those households who visited their financial intermediary in the past. Since these households can't reschedule their financial transactions, they must reduce their current consumption. As long as local households are keener on the consumption of domestic goods than their foreign counterparts this reduction in consumption is translated into a larger excess supply of domestic goods than of foreign goods. This, in turn, worsens the terms of trade.

On the other hand, under fixed exchange rates the terms of trade are unaffected by open market operations since these change the monetary holdings of the whole world's residents.

The paper is organized as follows: Section II presents the model while Section III derives its equilibrium. The effects of monetary policy under flexible exchange rates are presented in Section IV while those which prevail under fixed exchange rates are discussed in Section V. Section VI presents some conclusions.

II The Model

The model is basically a two country version of the model which is presented in more detail in Rotemberg (1982). Competitive firms in the home country produce good 1 while those of the foreign country produce good 2. Total output of good i at t depends, via a constant returns to scale production function on the amount of labor hired in the relevant country at t and on the amount of good i invested at t - 1. To abstract from imports of intermediaries only the local good is used in the local production of goods. Since an amount of labor which is normalized to equal one is supplied inelastically in both countries,

$$Q_{it} = f_i(K_{i\ t-1}) \qquad i = 1,2 \qquad (1)$$

where Q_{it} is output of good i at t, $K_{i\ t-1}$ is the amount of good i invested at t-1 while the f_i are increasing and concave functions. Workers are assumed to be paid their marginal product. Therefore the total amount of good i at t obtained from investing one unit of good i at t-1 is $f_i'(K_{i\ t-1})$ where primes denote first derivatives.

There is an even number of households in each country. At time t households in the home country are assumed to maximize the utility function given by:

$$V_t = \sum_{\tau=t}^{\infty} \rho^{\tau-t} [\alpha \ln C_{1\tau}^j + (1-\alpha) \ln C_{2\tau}^j] \qquad (2)$$

where $C_{1\tau}^j$ and $C_{2\tau}^j$ are the consumptions by household j at τ of goods 1 and 2 respectively while α and ρ are parameters between zero and one.

Instead the residents of the foreign country maximize the utility function given by

$$V_t^* = \sum_{\tau=t}^{\infty} \rho^{\tau-t} [\beta \ln C_{1\tau}^{*j} + (1-\beta) \ln C_{2\tau}^{*j}] \qquad (3)$$

where $C_{1\tau}^{*j}$ and $C_{2\tau}^{*j}$ are the consumptions at τ of household j while β is a parameter between zero and one.

The households at home have access to three assets, home money and claims on both capitals. There is thus perfect capital mobility. Local money is the only means of local exchange. Since money of the foreign country is rate of return dominated by foreign capital and cannot be used to carry out transactions in the home country it will never be held at home. Visits to the financial intermediary for the purpose of converting claims on capital into money are costly. Therefore, as in the models of Baumol (1952) and Tobin (1956), households find it optimal to visit their intermediaries only sporadically. As in Rotemberg (1982) I assume that households exchange capitals for money every two periods. The assumption that the timing of household's financial transactions is unresponsive to events is made for tractability. Except in stationary environments, the optimal timing of financial transactions is extremely difficult to find when households pick their consumption optimally.

Without loss of generality consider a household who engages in financial transactions at τ. At this time it withdraws an amount M_τ^τ of money balances. For simplicity I assume that wages are received in the form of claims on capital. Moreover, since money is rate of return dominated there is no reason for a household who withdraws money at τ to carry money over until period $\tau+2$. Thus, M_τ^τ must be equal to total expenditures at τ and $\tau+1$:

$$P_{1\tau}C^{\tau}_{1\tau} + P_{2\tau}C^{\tau}_{2\tau} + P_{1\tau+1}C^{\tau}_{1\tau+1} + P_{2\tau+1}C^{\tau}_{2\tau+1} = M^{\tau}_{\tau} \qquad (4)$$

where $P_{i\tau}$ is the home price of good i at τ while $C^{\tau}_{i\tau}$ and $C^{\tau}_{i\tau+1}$ are the consumptions of good i at τ and $\tau+1$ of a household that visits the financial intermediary at τ.

Once the household has picked optimally the sequence of monetary withdrawals it must choose consumption at τ and $\tau+1$ to maximize:

$$\alpha \ln C^{\tau}_{1\tau} + (1-\alpha) \ln C^{\tau}_{2\tau} + \rho\,\alpha \ln C^{\tau}_{1\tau+1} + \rho(1-\alpha) \ln C^{\tau}_{2\tau+1} \qquad (5)$$

subject to (4). This yields: [1]

$$\frac{\alpha}{1-\alpha} = \frac{P_{1\tau}C^{\tau}_{1\tau}}{P_{2\tau}C^{\tau}_{2\tau}} = \frac{P_{1\tau+1}C^{\tau}_{1\tau+1}}{P_{2\tau+1}C^{\tau}_{2\tau+1}} \qquad (6)$$

and

$$\frac{\rho\, P_{1\tau}\, C^{\tau}_{1\tau}}{P_{1\tau+1}C^{\tau}_{1\tau+1}} = 1 \qquad (7)$$

The relationship between $C^{\tau}_{i\tau}$ and $C^{\tau+2}_{i\tau+2}$ can be obtained by a simple perturbation argument. These are the consumptions at two consecutive dates in which the household visits the financial intermediary. Hence the household shouldn't be able to make itself better off by consuming one unit less of good i at τ, investing it in claims on capital of industry i, reinvesting the proceeds at $\tau+1$ in similar claims and consuming the proceeds at $\tau+2$. The reverse transaction also should not

increase utility. Hence:

$$\frac{\rho^2 \, C_{i\tau}^{\tau}}{C_{i\tau+2}^{\tau+2}} f_i'(K_{i\tau}) \, f_i'(K_{i\tau+1}) = 1 \qquad i = 1,2 \qquad (8)$$

These relationships which state that the product of the marginal rate of substitution between consumption in two periods times the return from holding an asset between the periods is equal to one have been derived by many authors including Grossman and Shiller (1981) and Hansen and Singleton (1981).

It is easy to verify that suitably modified versions of equations (6) - (8) hold for the foreign country as well. In particular:

$$\frac{\beta}{1-\beta} = \frac{P_{1\tau}^{*} \, C_{1\tau}^{\tau *}}{P_{2\tau}^{*} \, C_{2\tau}^{\tau *}} = \frac{P_{1\tau+1}^{*} \, C_{1\tau+1}^{\tau *}}{P_{2\tau+1}^{*} \, C_{2\tau+1}^{\tau *}} \qquad (6')$$

$$\frac{\rho \, P_{1\tau}^{*}}{P_{1\tau+1}} \cdot \frac{C_{1\tau}^{\tau *}}{C_{1\tau+1}^{\tau *}} = 1 \qquad (7')$$

$$\frac{\rho^2 \, C_{i\tau}^{\tau *}}{C_{i\tau+2}^{\tau+2 *}} f'(K_{i\tau}) \, f'(K_{i\tau+1}) = 1 \qquad i = 1,2 \qquad (8')$$

where $P_{i\tau}^{*}$ is the price of good i at τ in the foreign country's currency while $C_{i\tau+1}^{\tau *}$ is the consumption of good i at $\tau+1$ by a household of the foreign country who visits his intermediary at τ.

Costs of transporting goods between countries as well as tariffs and other trade barriers are neglected. Therefore, the relative price of the two goods at τ (or terms of trade) must be the same in both countries. In particular let R_τ denote the number of units of good 2 one can buy with one unit of good 1. Then:

$$R_\tau = \frac{P_{1\tau}}{P_{2\tau}} = \frac{P^*_{1\tau}}{P^*_{2\tau}} \quad .$$

Note that dividing (8) by (8') one obtains that:

$$\frac{C^\tau_{i\tau}}{C^{\tau*}_{i\tau}} = \frac{C^{\tau+2}_{i\tau+2}}{C^{\tau+2*}_{i\tau+2}} \qquad i = 1,2 \qquad (9)$$

But, since the terms of trade are the same in both countries (6) and (6') imply that:

$$\frac{C^\tau_{1\tau}}{C^\tau_{2\tau}} = \frac{C^{\tau*}_{1\tau}}{C^{\tau*}_{2\tau}} \; / \; \phi \qquad \phi = \frac{\beta(1-\alpha)}{\alpha(1-\beta)} \qquad (10)$$

Therefore, if $C^\tau_{1\tau}/C^{\tau*}_{1\tau}$ is equal to k_τ then $C^\tau_{2\tau}/C^{\tau*}_{2\tau}$ is equal to ϕk_τ. Moreover (9) implies that k_τ is equal to some constant, say k_e, for τ even and to another constant k_o for τ odd. The k's themselves are determined by the relative wealths of the residents of the home and foreign countries. Loosely speaking (9) and (10) state that the consumptions of the residents of the two nations are proportional. The factor of proportionality thus depends on which country is richer.

This fact is illustrated in the Appendix. In the remainder of the paper I assume for simplicity that k_e and k_o are equal to k and I neglect the effects of monetary policy on these k's. Stockman (1980) also ignores these effects.

The intermediaries in both countries hold in the name of the households the private sectors' claims on the two capitals. In addition the intermediaries are allowed to issue a certain quantity of money. In particular, the monetary liabilities of the home country's intermediaries between τ and $\tau+1$ must be equal to the quantity of outside money (or deposits at the Fed) given by M_τ. Similarly, the monetary liabilities of the foreign intermediaries between τ and $\tau+1$ must equal M^*_τ. Since households are allowed to spend at τ the money they withdraw at τ, these assumptions ensure that not all of either country's households visit their intermediaries the same day. This is so since if all households in the home country visited the intermediary at τ no one would hold money between $\tau+1$ and $\tau+2$. Here, it is assumed that half the households in both countries visit their intermediaries at τ while the other half visits theirs at $\tau+1$. The fact that different people perform financial transactions at different times is the crucial feature of reality the models of Rotemberg (1982) and this paper seek to capture. As suggested in the introduction, this fact seems to be crucial in ensuring that monetary policy affects the terms of trade.

The governments of this model have no expenditures. However, they levy taxes, issue local money and hold claims on capital. The evolution of the capital stocks held by the government is given by:

$$K^{G}_{1\tau+1} = f'_1(K_{1\tau})\,K^{G}_{1\tau} - \frac{1}{R_\tau}\left[K^{G}_{2\tau+1} - f'_2(K_{2\tau})\,K^{G}_{2\tau}\right] + \frac{M_{\tau+1}-M_\tau}{P_{\tau+1}} + T_{\tau+1}$$

$$K^{G*}_{1\tau+1} = f'_1(K_{1\tau})\,K^{G*}_{1\tau} - \frac{1}{R_\tau}\left[K^{G*}_{2\tau+1} - f'_2(K_{2\tau})\,K^{G}_{2\tau}\right] + \frac{M^{*}_{\tau+1}-M^{*}_\tau}{P_{\tau+1}} + T_{\tau+2} \qquad (11)$$

Here $K^{G}_{i\tau}$ and $K^{G*}_{i\tau}$ are the capitals of type i held at τ by the home and foreign governments respectively while T_τ and T^{*}_τ are the lump sum taxes levied by the home and foreign governments at τ. Without loss of generality these taxes are levied in the form of capital of sector 1. An increase in money which is used to buy capital is called an open market purchase, and is the domain of monetary policy. Instead swaps of $K^{G}_{1\tau}$ for $K^{G}_{2\tau}$ or of $K^{G*}_{1\tau}$ for $K^{G*}_{2\tau}$ are foreign exchange market interventions. These interventions have no effect in this model.

III Equilibrium Under Flexible Exchange Rates

Under flexible exchange rates the governments of the two countries set the paths of M_τ and M^{*}_τ. An equilibrium is a path for the terms of trade, the prices of good 1 in both countries, and the two real rates of interest such that:

a) The sum of consumptions and investment of good i demanded by households and governments at τ is equal to the output of good i at τ:

$$C_{i\tau} = N(C_{i\tau}^{\tau} + C_{i\tau}^{\tau-1}) + N^{*}(C_{i\tau}^{\tau*} + C_{i\tau}^{\tau-1*}) = f_i\,(K_{i\tau-1}) - K_{i\tau} \quad i=1,2 \qquad (12)$$

where $C_{i\tau}$ is total consumption of good i at τ while 2N and $2N^{*}$ are the populations of the home and foreign country respectively.

b) The amounts of money that households who visit their intermediary at τ wish to hold between τ and $\tau+1$ in the home and foreign country must be equal to M_{τ} and M_{τ}^{*} respectively.

Condition b) requires that:

$$\begin{aligned} N\,[P_{1\tau}\,C_{1\tau+1}^{\tau} + P_{2\tau}\,C_{2\tau+1}^{\tau}] &= M_{\tau} \\ N^{*}[P_{1\tau}^{*}\,C_{1\tau+1}^{\tau*} + P_{2\tau}^{*}\,C_{2\tau+1}^{\tau*}] &= M_{\tau}^{*} \quad . \end{aligned} \qquad (13)$$

Since households who visit the intermediary at τ maximize $\alpha \ln C_{1\tau+1}^{\tau} + (1-\alpha) \ln C_{2\tau+1}^{\tau}$ subject to (13) in equilibrium:

$$\begin{aligned} P_{1\tau+1}\,C_{1\tau+1}^{\tau} &= \alpha\,\frac{M_{\tau}}{N} \\ P_{2\tau+1}\,C_{2\tau+1}^{\tau} &= (1-\alpha)\,\frac{M_{\tau}}{N} \end{aligned} \qquad (14)$$

On the other hand, using (7) and (14):

$$\begin{aligned} \rho\,P_{1\tau+1}\,C_{1\tau+1}^{\tau+1} &= \alpha\,\frac{M_{\tau+1}}{N} \\ \rho\,P_{2\tau+1}\,C_{2\tau+1}^{\tau+1} &= (1-\alpha)\,\frac{M_{\tau+1}}{N} \end{aligned} \qquad (15)$$

Hence:

$$C^{\tau}_{i\tau+1} = \rho \frac{M_\tau}{M_{\tau+1}} C^{\tau+1}_{i\tau+1} \quad . \qquad i = 1,2 \qquad (16)$$

Using the same argument it is straightforward to establish that:

$$C^{\tau*}_{i\tau+1} = \rho \frac{M^*_\tau}{M^*_{\tau+1}} C^{\tau+1*}_{i\tau+1} \qquad i = 1,2 \qquad (17)$$

Using the proportionality of $C^{\tau}_{i\tau}$ and $C^{\tau*}_{i\tau}$, total consumption of good i at τ is given by:

$$\begin{aligned} C_{1\tau} &= N\, C^{\tau}_{1\tau} \left[1 + \rho \frac{M_\tau}{M_{\tau+1}} + \frac{N^*}{N\bar{k}} \left[1 + \rho \frac{M^*_\tau}{M^*_{\tau+1}}\right]\right] \\ C_{2\tau} &= N\, C^{\tau}_{2\tau} \left[1 + \rho \frac{M_\tau}{M_{\tau+1}} + \frac{N^*}{N\phi\bar{k}} \left[1 + \rho \frac{M^*_\tau}{M^*_{\tau+1}}\right]\right] \end{aligned} \qquad (18)$$

Therefore, using (8) and the equilibrium condition (12):

$$f_1(K_{1\tau+1}) - K_{1\tau+2} = \rho^2 \frac{1 + \rho\frac{M_{\tau+1}}{M_{\tau+2}} + \bar{k}\left(1 + \rho\frac{M^*_{\tau+1}}{M^*_{\tau+2}}\right)}{1 + \rho\frac{M_{\tau-1}}{M_\tau} + \bar{k}\left(1 + \rho\frac{M^*_{\tau-1}}{M^*_\tau}\right)} f_1'(K_{1\tau})\, f_1'(K_{1\tau+1}) \qquad (19)$$

$$[f_1(K_{1\tau-1}) - K_{1\tau}]$$

$$\tau = t, t+1, \ldots .$$

$$f_2(K_{2\tau+1}) - K_{2\tau+2} = \rho^2 \frac{1 + \rho \frac{M_{\tau+1}}{M_{\tau+2}} + \frac{\bar{k}}{\phi}\left(1 + \rho \frac{M^*_{\tau+1}}{M^*_{\tau+2}}\right)}{1 + \rho \frac{M_{\tau-1}}{M_\tau} + \frac{\bar{k}}{\phi}\left(1 + \rho \frac{M^*_{\tau-1}}{M^*_\tau}\right)} f_2'(K_{2\tau}) \, f_2'(K_{2\tau+1}) \tag{20}$$

$$[f_2(K_{2\tau-1}) - K_{2\tau}]$$

$$\tau = t, t+1, \ldots .$$

where $\bar{k} = \frac{N^*}{kN}$.

Equations (19) and (20) are very similar to the equation which characterizes the closed economy equilibrium of Rotemberg (1982). The knowledge of the sequence of capitals obtained from these equations directly yields the sequence of rates of return. The sequence of aggregate consumptions can be obtained from (12) while the sequence of individual consumptions follows from (10), (16), (17) and knowledge of k. Finally, the sequence of prices can be computed using (15) and the equivalent relationships for the foreign country.

Note that (19) and (20) are uncoupled difference equations in $K_{1\tau}$ and $K_{2\tau}$. This lack of coupling reflects the considerable simplification of assuming that only good i is required to produce good i. These equations are nonlinear third order difference equations with only one initial condition, namely K_{it-1}. Hence they have infinitely many solutions which are indexed by K_{it} and K_{it+1}, the two "arbitrary" initial conditions. As explained in Rotemberg (1982) there are two arbitrary conditions because both C^t_{it} and C^{t+1}_{it+1} can be chosen at will.

On the other hand, as will be apparent below, near the steady state there is only one equilibrium path which is nonexplosive and which has no arbitrary oscillations in the steady state. Before proceeding with the linearization of (19) and (20) which establishes this fact, it is apparent from these equations that certain government interventions are neutral while others are not.

Consider as the base case a situation in which money is constant in both countries. Then (19) and (20) become:

$$f(K_{1\tau+1}) - K_{1\tau+2} = \rho^2 f_1'(K_{1\tau}) f_1'(K_{1\tau+1}) [f(K_{1\tau-1}) - K_{1\tau}] \qquad (19')$$

$$f(K_{2\tau+1}) - K_{2\tau+2} = \rho^2 f_2'(K_{2\tau}) f_2'(K_{2\tau+1}) [f(K_{2\tau-1}) - K_{2\tau}] \qquad (20')$$

Consider a particular set of equilibrium paths for $K_{1\tau}$ and $K_{2\tau}$ given by $\{\hat{K}_{1\tau}\}$ and $\{\hat{K}_{2\tau}\}$. Suppose either government engages in exchange market interventions at t. These do not affect (19) or (20) and hence $\{\hat{K}_{1\tau}\}$ and $\{\hat{K}_{2\tau}\}$ are still equilibrium paths. The neutrality of these interventions is due to the fact that the two capitals are perfect substitutes in households' portfolios.[2]

Suppose instead that starting at t, the rate of growth of either money stock unexpectedly increases to a new constant higher level. Then, as long as M_t / M_{t-1} is still equal to M_{t+i} / M_{t+i-1} and M_t^* / M_{t-1}^* is still equal to M_{t+i}^* / M_{t+i-1}^*, equations (19') and (20') still represent the equilibrium. Hence such increases in the rate of monetary growth are consistent with $\{\hat{K}_{1\tau}\}$ and $\{\hat{K}_{2\tau}\}$; they are neutral. This must be contrasted with the money-in-the-utility-function model of Obstfeld (1981) in which changes in the growth rate of money have real effects.

Instead, again in contrast to Obstfeld's (1981) model, open market operations have real effects even when, as here, the government earns interest on its holdings of claims on both domestic and foreign capital. This can be seen by considering the effects of increasing M_t (decreasing M_{t-1} / M_t) and leaving unchanged all other ratios of the form M_{t+i} / M_{t+i-1}. Then suppose that $\hat{K}_{1t}$, $\hat{K}_{1t+1}$, $\hat{K}_{2t}$ and $\hat{K}_{2t+1}$ are still equilibrium levels of the capital stocks. Then by (19) and (20), $\hat{K}_{1t+2}$ and $\hat{K}_{2t+2}$ will not be equilibrium levels of the capital stocks. The open market purchase affects capital and hence output. The actual effects of small open market operations near the steady state will be considered in the next section. First the equations (19) and (20) need to be linearized around the steady state values of $K_{1\tau}$ and $K_{2\tau}$. As shown in Rotemberg (1982) there is a unique steady state in which both production and consumption of the two goods is positive.[3] This steady state is given by $\bar{K}_1$ and $\bar{K}_2$ where

$$\rho f_i'(\bar{K}_i) = 1 \qquad i = 1, 2 \qquad (21)$$

Moreover, in the steady state it is assumed that monetary growth is constant in both countries. Then, using (21) and the linearization of (19) and (20) around $\bar{K}_1$ and $\bar{K}_2$ yields:[4]

$$(K_{1\tau+2} - \bar{K}_1) - \{f_1'(\bar{K}_1) - \rho\, f_1''(\bar{K}_1)\,(f_1(\bar{K}_1) - \bar{K}_1)\}\,(K_{1\tau+1} - \bar{K}_1)$$

$$- \{1 - \rho\, f_1''(\bar{K}_1)\,(f_1(\bar{K}_1) - \bar{K}_1)\}\,(K_{1\tau} - \bar{K}_1) + f_1'(\bar{K}_1)(K_{1\tau} - \bar{K}_1)$$

$$= -(f_1(\bar{K}_1) - \bar{K}_1)\left[\frac{1 + \rho\,\frac{M_{\tau+1}}{M_{\tau+2}} + \bar{k}\left(1 + \rho\,\frac{M^*_{\tau+1}}{M^*_{\tau+2}}\right)}{1 + \rho\,\frac{M_{\tau-1}}{M_\tau} + \bar{k}\left(1 + \rho\,\frac{M^*_{\tau-1}}{M^*_\tau}\right)} - 1\right] \tag{22}$$

$$(K_{2\tau+2} - \bar{K}_2) - \{f_2'(\bar{K}_2) - \rho\, f_2''(\bar{K}_2)\,(f_2(\bar{K}_2) - \bar{K}_2)\}\,(K_{2\tau+2} - \bar{K}_2)$$

$$- \{1 - \rho\, f_2''(\bar{K}_2)\,(f_2(\bar{K}_2) - \bar{K}_2)\}\,(K_{2\tau+1} - \bar{K}_2)$$

$$+ f_2'(\bar{K}_2)\,(K_{2\tau} - \bar{K}_2) = -(f_2(\bar{K}_2) - \bar{K}_2) \tag{23}$$

$$\left[\frac{1 + \rho\,\frac{M_{\tau+1}}{M_{\tau+2}} + \frac{\bar{k}}{\phi}\left(1 + \rho\,\frac{M^*_{\tau+1}}{M^*_{\tau+2}}\right)}{1 + \rho\,\frac{M_{\tau-1}}{M_\tau} + \frac{\bar{k}}{\phi}\left(1 + \rho\,\frac{M^*_{\tau-1}}{M^*_\tau}\right)} - 1\right]$$

These equations can be rewritten as:

$$(1 - \lambda_i L)\,(1 - \mu_i L)\,(1 - \gamma_i L)\,K_{i\tau+2} = \theta_i + F_{i\tau+2}$$

$$i = 1,\ 2 \tag{24}$$

where L is the lag operator and:

$$\theta_i = (2\bar{K}_i \rho f_i''(\bar{K}_i))(f_i(\bar{K}_i) - \bar{K}_i) \qquad i = 1, 2$$

$$F_{1\tau} = -\left[\frac{1 + \rho \frac{M_{\tau-1}}{M_\tau} + \bar{k}\left(1 + \rho \frac{M^*_{\tau-1}}{M^*_\tau}\right)}{1 + \rho \frac{M_{\tau-3}}{M_{\tau-2}} + \bar{k}\left(1 + \rho \frac{M^*_{\tau-3}}{M^*_{\tau-2}}\right)} - 1\right](f_1(\bar{K}_1) - \bar{K}_1)$$

$$F_{2\tau} = -\left[\frac{1 + \rho \frac{M_{\tau-1}}{M_\tau} + \frac{\bar{k}}{\phi}\left(1 + \rho \frac{M^*_{\tau-1}}{M^*_\tau}\right)}{1 + \rho \frac{M_{\tau-3}}{M_{\tau-2}} + \frac{\bar{k}}{\phi}\left(1 + \rho \frac{M^*_{\tau-3}}{M^*_{\tau-2}}\right)} - 1\right](f_2(\bar{K}_2) - \bar{K}_2)$$

$$\lambda_i \mu_i \gamma_i = -f_1'(\bar{K}_i)$$

$$\lambda_i \mu_i + \mu_i \gamma_i + \lambda_i \gamma_i = -1 + \rho f_i''(\bar{K}_i)(f_i(\bar{K}_i) - \bar{K}_i)$$

$$\lambda_i + \mu_i + \gamma_i = f_i'(\bar{K}_i) - \rho f_i''(\bar{K}_i)(f_i(\bar{K}_i) - \bar{K}_i)$$

Inspection of these last three equations establishes that one of the roots, say γ_i is equal to minus one. Instead, $(\lambda_i - 1)(1 - \gamma_i)$ is positive as are $\lambda_i\gamma_i$ and $(\lambda_i + \gamma_i)$. Therefore λ_i and γ_i are both positive and while one of them (say λ_i) is less than one the other is larger than one. Blanchard and Kahn (1980) establish that for a unique nonexplosive solution to (24) to exist, the number of roots at or inside the unit circle must

be equal to the number of predetermined variables. Here two roots are at or inside the unit circle and there is only one predetermined variable (K_{it-1}). There thus exist an infinity of nonexplosive paths. However, since one of the roots is equal to minus one, there exists only one path which does not oscillate in the steady state. This equilibrium path can be found using techniques analogous to those of Blanchard and Kahn (1980):

$$K_{i\tau} = \lambda_i K_{i\tau-1} - \frac{1}{1+\mu_i} \sum_{j=0}^{\infty} \left[\left(\frac{1}{\mu_i}\right)^j - (-1)^j\right] K_{i\tau+1+j} + (1-\lambda_i) \bar{K}_i \qquad (25)$$

$$\tau = t, t+1, \ldots.$$

Note that the coefficient of F_{it+1} is zero in (25). Moreover as long as money in both countries grows at the same rate at t as it does in future periods, F_{it+1+j} is zero. For this case equation (25) asserts that the capitals converge at the rates λ_i to their steady state values. This speed of convergence differs across countries if λ_1 is different from λ_2. Similarly the impact of monetary shocks differs if μ_1 is different from μ_2. Hence, given the relationships below (24), nontrivial dissimilarities in the technologies of the two countries are translated into differences in the dynamic behavior of their capital stocks. On the other hand, differences in the population of the two countries do not affect the roots λ_i and μ_i.

Even when the technologies of the two countries are identical differences in tastes can insure that the effect of a monetary shock is different in the two countries. This is studied in the next section.

IV Monetary Policy and the Terms of Trade

Suppose the two countries have identical technologies so that μ_i is equal to μ while λ_i is equal to λ. Now consider a once and for all open market purchase of capital at t which takes place only in the home country. This induces a once and for all increase in M_t. This reduces F_{1t+2} and F_{2t+2} leaving all F's beyond t+2 unaffected. Note that unless ϕ is equal to one the effect on F_{1t+2} is different from the effect on F_{2t+2}. Hence the effect on the two capitals is different unless ϕ is one. ϕ is equal to one if and only if α is equal to β. If households in the home country have a more pronounced preference for home goods than do households in the foreign country α exceeds β and ϕ is smaller than one. This is the case that will be considered in this section. When ϕ is smaller than one the fall in F_{1t+2} after a monetary expansion at home is bigger than the fall in F_{2t+2}. The coefficient of F_{it+2} in the equation (25) which gives K_{it} is $-1/\mu$ which is negative. Therefore an increase in the money supply at t raises capital at home and raises it by more than it raises capital abroad. The intuition behind this result is straightforward. An increase in M_t raises prices at home. Therefore it reduces the real value of the purchases made by residents of the home country who visited the bank at t-1. Since these residents buy relatively more home goods, the consumption of home goods falls relative to the consumption of foreign goods. In this equilibrium model what isn't consumed is invested. Thus there is more capital accumulation in the home country. In turn, this raises output in the home country and this increase is, again, larger than the increase in the foreign country. In any event the flexibility of exchange rates does not insulate the foreign country from our monetary

injection.

Since capitals and outputs are differentially affected it appears likely that the terms of trade respond to monetary expansions. To see that this is so, using (6) and (18) the terms of trade at t can be written as:

$$R_t = \frac{\alpha}{1-\alpha} \frac{C_{1t}}{C_{2t}} \left(\frac{1 + \rho \frac{M_t}{M_{t+1}} + \bar{k} \left(1 + \rho \frac{M_t^*}{M_{t+1}^*} \right)}{1 + \rho \frac{M_t}{M_{t+1}} + \frac{\bar{k}}{\phi} \left(1 + \rho \frac{M_t^*}{M_{t+1}^*} \right)} \right) \qquad (26)$$

The term in brackets is unaffected by the monetary expansion. Instead, both levels of capital at t are affected and therefore the ratio of consumption C_{1t} / C_{2t} is likely to change inducing a change in the terms of trade. Suppose in particular that at t-1 both capitals were equal to their steady state values. Also suppose that, before the monetary expansion, K_{it} was expected also to equal $\bar{K}$, the common steady state value of capital. Then the monetary expansion increases K_{1t} relative to K_{2t} and thus reduces the ratio C_{1t} / C_{2t} . It leads to a real depreciation. Intuitively, when α exceeds β the monetary expansion leads, as prices rise at home to a larger excess supply of good 1 than the excess supply of good 2. This puts downward pressure on the relative price of good 1. The monetary expansion also leads to a nominal depreciation. The nominal exchange rate at t, e_t, is given by P_{1t} / P_{1t}^* . Using (19) and the equivalent relationship for the other country:

$$e_t = \frac{\alpha N^* \, C_{1t}^{t-1*} \, M_t}{\beta N \, C_{1t}^{t-1} \, M_t^*} \qquad (27)$$

Employing the proportionality of C_{1t}^{t} and C_{1t}^{t*} as well as (16) and (17):

$$e_t = k \frac{\alpha N^* (M_t)^2 M_{t-1}^*}{\beta N (M_t^*)^2 M_{t-1}} \tag{28}$$

Therefore money is a central determinant of the exchange rate. The home exchange rate depreciates as money at home rises relative to money abroad and relative to money the previous period. Also the exchange rate depreciates as the population at home falls relative to the population abroad and as the home country increases its taste for home produced goods.

V Equilibrium Under Fixed Exchange Rates

Let the exchange rate be set at e so that $P_{1t} = e P_{1t}^*$. This section establishes mainly that the terms of trade effects of monetary policy are absent in a regime of fixed exchange rates when the technologies of the two countries are identical. This is so because an increase in either country's money simply leads to an increase in the world money supply. Hence all households who went to the bank the period before the increase in the money supply must reduce their consumption. So, neither good is in more excess supply than the other and their relative price need not change.

Under fixed exchange rates the governments do not control their local money supply. Instead, their collective open market operations only fix $M_t + e M_t^*$ which will be called $\bar{M}_t$, the world money supply measured in home currency. This has been established by numerous authors including Swoboda (1978). Hence equilibrium under fixed exchange rates

requires in addition to the conditions a) and b) of the equilibrium under flexible exchange rates that:

c) The sum of the money valued in home currency that all households who visit their intermediaries at τ want to carry over to $\tau+1$ must equal $\bar{M}_\tau$.

The analysis of the model is considerably simplified by the fact that one of its implications is that the fraction of $\bar{M}_\tau$ held by domestic residents is constant. Since e_t is constant (27) becomes:

$$e = \frac{\alpha N^* \; M_\tau \; C_{1\tau}^{\tau-1*}}{\beta N \; M_\tau^* \; C_{1\tau}^{\tau-1}} \tag{29}$$

On the other hand, using (7) and (7'):

$$\frac{C_{1\tau}^{\tau-1*}}{C_{1\tau}^{\tau-1}} = \frac{P_{1\tau-1}^* \; P_{1\tau} \; C_{1\tau-1}^{\tau-1*}}{P_{1\tau}^* \; P_{1\tau-1} \; C_{1\tau-1}^{\tau-1}} = \frac{1}{k} \tag{30}$$

Where the last equality follows from the fact that exchange rates don't change between $\tau-1$ and τ and from the proportionality of $C_{1\tau}^{\tau}$ and $C_{1\tau}^{\tau*}$. Hence:

$$M_\tau = \frac{k\beta N}{\alpha N^*} \; e \; M_\tau^* \tag{31}$$

and:

$$M_\tau = \frac{\alpha N^*}{\alpha N^* + k\beta N} \bar{M}_t$$
$$M^*_\tau = \frac{k\beta N}{e\,(\alpha N^* + k\beta N)} \bar{M}_t \qquad (32)$$

so that:

$$\frac{M_\tau}{M_{\tau-1}} = \frac{M^*_\tau}{M^*_{\tau-1}} = \frac{\bar{M}_\tau}{\bar{M}_{\tau-1}} \qquad (33)$$

Since households at home who visit their intermediaries at τ must still carry over M_τ units of money from τ to $\tau+1$ and similarly for foreign households, equations (19) and (20) remain valid in equilibrium. Thus, using (33):

$$f_1(K_{1\tau+1}) - K_{1\tau+2} = \rho^2 \frac{1 + \rho \dfrac{\bar{M}_{\tau+1}}{\bar{M}_{\tau+2}}}{1 + \rho \dfrac{\bar{M}_{\tau-1}}{\bar{M}_\tau}} f_1'(K_{1\tau})\, f_1'(K_{1\tau+1})\, [f_1(K_{1\tau-1}) - K_{1\tau}] \qquad (34)$$

$$f_2(K_{2\tau+1}) - K_{2\tau+2} = \rho^2 \frac{1 + \rho \dfrac{\bar{M}_{\tau+1}}{\bar{M}_{\tau+2}}}{1 + \rho \dfrac{\bar{M}_{\tau-1}}{\bar{M}_\tau}} f_2'(K_{2\tau})\, f_2'(K_{2\tau+1})\, [f_2(K_{2\tau-1}) - K_{2\tau}] \qquad (35)$$

So, if the technologies of the two goods are identical and K_{1t-1} is equal to K_{2t-1}, a monetary change at t in either country has the same

effect on the consumptions of the two goods at t. Therefore, by (26) the terms of trade are unaffected by the monetary change.

VI Conclusions

This paper has presented a simple two country equilibrium model in which money is held for transaction purposes. While the model is very simple it is able to give a monetary explanation for the substantial fluctuations in the terms of trade that have characterized the floating exchange rate period of the 1970's. The model also predicts that monetary forces induce fewer fluctuations in the terms of trade under fixed exchange rates. It must be noted that previous models with flexible prices like those of Stockman (1980) and Obstfeld (1981) give no explanation for the higher variance of the terms of trade under flexible exchange rates. On the other hand, models with sticky prices like the one in Dornbusch (1976) can also give a monetary explanation for this variability. It is an open empirical question which of sticky prices and staggered visits to financial intermediaries is more important in generating the variability of the terms of trade.

The model of this paper can be extended in several directions. First, it can be used to analyze both the steady state and the non-steady state effects commercial policies as well as of devaluations. Second, it may be able to shed light on the effect of various monetary institutions on the behavior of the balance of payments and the terms of trade. Monetary institutions of interest include commodity standards and various intervention rules.

FOOTNOTES

[1]Interior solutions to the households maximization problem are assumed throughout.

[2]However, the Modigliani-Miller arguments of Chamley and Polemarchakis (1982) make it likely that this neutrality would prevail even if the two capitals had different stochastic returns.

[3]This will be the only steady state considered below. There are two other steady states. One has zero capital and hence zero output and consumption. The other has zero consumption because investment is equal to output. This latter steady state isn't consistent with individual rationality since households can make themselves better off by running down their wealth.

[4]In the expressions below double primes denote second derivatives.

APPENDIX

The purpose of this appendix is to show that k_e and k_o depend on the relative wealths of the two countries. The total monetary withdrawal at τ, M_τ^τ, of a household who visits the intermediary at τ is given by $P_{1\tau}C_{1\tau}^\tau + P_{2\tau}C_{2\tau}^\tau + P_{1\tau+1}C_{1\tau+1}^\tau + P_{2\tau+1}C_{2\tau+1}^\tau$. Therefore, using (6) and (7):

$$M_\tau^\tau = P_{1\tau}C_{1\tau}^\tau \left(\frac{1+\rho}{\rho\alpha}\right) \tag{A1}$$

Similarly for foreign households:

$$M_\tau^{\tau*} = P_{1\tau}^* C_{1\tau}^{\tau*} \left(\frac{1+\rho}{\rho\beta}\right) . \tag{A2}$$

The evolution of the capital holdings of the domestic household who visits his intermediaries at τ is given by:

$$K_{1\tau}^j + \frac{K_{2\tau}^j}{R_\tau} = f_1'(K_{1\tau-2})\, f_1'(K_{1\tau-1})\, K_{1\tau-2}^j + f_2'(K_{2\tau-2})\, f_2'(K_{2\tau-1})\, \frac{K_{2\tau-2}^j}{R_\tau} + Y_\tau^j - \frac{M_\tau^\tau}{P_{1\tau}} \tag{A3}$$

where Y_τ^j is noncapital income of household j at τ, which, for simplicity is assumed to accrue in the form of capital of type 1. Note that (A3) assumes that the proceeds from investments in capital of type 1 at $(\tau-2)$ are reinvested in capital of type 1 and similarly for investments in

capital of type 2. Using (6) and (8):

$$R_\tau = \frac{f_2'(K_{2\tau-2})\ f_2'(K_{2\tau-1})}{f_1'(K_{1\tau-2})\ f_1'(K_{1\tau-1})}\ R_{\tau-2} \qquad (A4)$$

which is the "real" version of uncovered interest arbitrage. Therefore (A3) becomes:

$$K^j_{1\tau} + \frac{K^j_{2\tau}}{R_\tau} = f_1'(K_{1\tau-2})\ f_1'(K_{1\tau-1})\ [K^j_{1\tau-2} + \frac{K^j_{2\tau-2}}{R_{\tau-2}}] + Y^j_\tau - \frac{M^\tau_\tau}{P_{1\tau}} \qquad (A5)$$

and, using the natural constraint which avoids explosive paths of debt:

$$\lim_{\tau\to\infty} \frac{K^j_{1\tau} + K^j_{2\tau} / R_\tau}{\prod_{h=t}^{\tau} f_1'(K_{1h-1})} = 0 \qquad (A6)$$

Therefore, using (A1), the lifetime budget constraint for a household who visits the intermediary at t becomes:

$$\frac{1+\rho}{\rho\alpha}\left\{ C^t_{1t} + \sum_{i=1}^{\infty} \frac{C^{t+2i}_{1t+2i}}{\prod_{h=1}^{i} f_1'(K_{1t+2h-2})\ f_1'(K_{1t+2h-1})} \right\}$$

$$= Y_t + f_1'(K_{1t-2})\ f'(K_{1t-1})\ (K^j_{1t-2} + \frac{K^j_{2t-2}}{R_{t-2}})$$

$$+ \sum_{i=1}^{\infty} \frac{Y_{t+i}}{\prod_{h=0}^{i-1} f_1'(K_{1t+h})} = W^t_t \qquad (A7)$$

where W^t_t is the total wealth at t of households who visit the intermediary at t.

Similarly for the foreign households:

$$\frac{1+\rho}{\rho\beta}\left[\, C_{1t}^{t*} + \sum_{i=1}^{\infty} \frac{C_{1t+2i}^{t+2i*}}{\prod_{h=1}^{i} f_1'(K_{1t+2h-2})\, f_1'(K_{1t+2h-1})} \,\right]$$

$$= Y_t^* + f_1'(K_{1t-2})\, f_1'(K_{1t-1})\left(K_{1t-2}^{j*} + \frac{K_{2t-2}^{j*}}{R_{t-2}}\right) + \sum_{i=1}^{\infty} \frac{Y_{t+i}^*}{\prod_{R=0}^{i-1} f_1'(K_{1t+h})}$$

$$= W_t^{t*} \qquad \text{(A8)}$$

where Y_τ^* is the noncapital income at τ of the foreign household while W_t^{t*} is its wealth at t if it visits the intermediary at t. Therefore, since for t even, k_e is given by C_{1t}^{t} / C_{1t}^{t*} while k_o is given by $C_{1t+1}^{t+1} / C_{1t+1}^{t+1*}$, the k's are given by:

$$k_e = \frac{\alpha\, W_t^t}{\beta\, W_t^{t*}} \qquad k_o = \frac{\alpha\, W_{t+1}^{t+1}}{\beta\, W_{t+1}^{t+1*}}$$

So, as claimed, the k's depend on the relative wealths.

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